Derivation of Hillslope Sediment Transport for PIHMSed

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To understand the feedbacks between regolith transport, thickness, and weathering in the SHO, we need to i) characterize rates of erosion throughout the watershed and ii) relate these to distributions of hillslope topography and regolith thickness through a landscape evolution model. Central to the development of a model to predict regolith evolution in the SHO is the quantitative understanding of the rates and processes of soil transport on hillslopes. Here I derive the set of equations describing the 1-D time-dependent evolution of the ground elevation and thickness of the regolith as functions of rock weathering, rock uplift, and hillslope sediment transport processes. The equations can easily be extended to 2-D plan view for incorporation into PIHMSed. It differs from (Li 2008) in that he considered only sediment transport by overland flow. This derivation contributes to Tasks 2.1.5, 2.1.6, and 2.1.8.

Consider the hillslope in Figure 1 where the variables are defined as in the caption. We want to predict the evolution of the ground elevation and regolith thickness as functions of location (x) (units of meters) and time (t) (measured in years to be consistent with standard practice). Let the regolith bulk density be σ_s and the rock bulk density be σ_r.

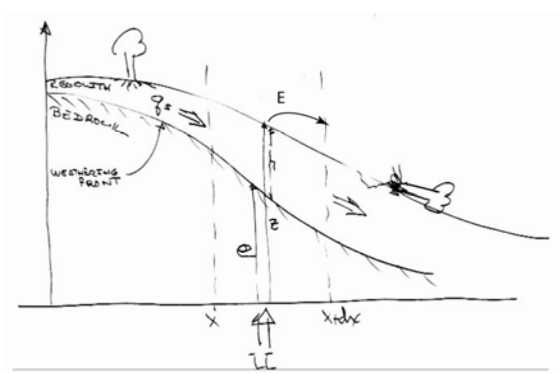

Figure 1. Definition sketch of hillslope where *z* = ground surface elevation (m), *e* = bedrock interface elevation (m), *h* = regolith thickness in vertical (m), U = rock uplift rate (positive upwards) (m yr^{-1}), *E* = the net erosion rate (negative for deposition) (m yr^{-1}) on the surface by overland flow, and qs_x = the volumetric regolith flux rate (positive in the x-dir) (m^3 m^{-1} yr^{-1}) entering the sides of the control volume.

Then from conservation of rock and regolith in the control volume:

$$\sigma_r \frac{\partial e}{\partial t} + \sigma_s \frac{\partial h}{\partial t} = -\sigma_s \frac{\partial qs_x}{\partial x} + \sigma_r U - \sigma_s E \tag{1.1}$$

where terms are defined in figure caption 1. Since $h = z - e$,

$$\frac{\partial z}{\partial t} = -(\frac{\sigma_r}{\sigma_s} - 1)\frac{\partial e}{\partial t} - \frac{\partial qs_x}{\partial x} + \frac{\sigma_r}{\sigma_s} U - E \tag{1.2}$$

The first term in (1.2) is the time rate of change of the ground surface with respect to a datum. The second term describes the motion of the bedrock/regolith interface due to bedrock weathering. In the literature (Dietrich, Bellugi et al. 2003; Dietrich and Perron

2006) there are a number of expressions for defining the bedrock weathering rate (second term) in Eqn. (1.2), but the consensus function is:

$$\frac{\partial e}{\partial t} = -P_o e^{-\alpha h} \tag{1.3}$$

where P_o = regolith production rate in the absence of soil (48 g m^2 yr^{-1} or about 4 x 10^{-6} m/yr or for SHO in Jin et al., submitted) and α is a fitting constant (m^{-1}) which is taken by Heimseth, Dietrich et al. (1997) to equal 0.022 m^{-1}. Other researchers (Ahnert 1976) have argued that regolith production is maximized not on bare bedrock but under a thin cover of soil which retains moisture, thereby requiring a parabolic function in place of (1.3). Here we retain the simpler function. Notice that the ground surface rises as weathering proceeds due to the volumetric expansion as rock is converted in soil.

Lateral volumetric regolith flux

The third term in (1.2) represents all processes moving regolith laterally through the control volume sides. This lateral volumetric regolith flux qs_x, involves a number of sediment transport processes: 1) creep mechanisms as for example by cryoturbation (freeze-thaw) and wetting and drying; 2) slow bulk downslope sliding such as solifluction (not considered here at present); 3) plant root decay; 4) tree throw; and 5) asymmetric bioturbation by various animals (not considered here). Here I am making the distinction between regolith flux through the sides of the cell and surface sediment transport flux which is treated later.

Creep Mechanisms

Creep is typically modeled as a diffusion process wherein:

$$qs_x = -D\frac{\partial z}{\partial x}.$$

and D is the diffusivity (m^2 yr^{-1}). Studies in coastal California and the Wind River Range of Wyoming cited in Dietrich, Belugi, et al. (2003) show surprising confirmation that creep flux is linear with slope, but the diffusivities vary by a factor of 2, probably due to differing dominant mechanisms at the two sites. [We should attempt to use our dating studies to get a D for SHO, IF we can separate it from the other processes. One would expect that D would differ between the two side slopes at SHO due to aspect.].

Plant Root Decay

The flux due to plant root decay arises on slopes because the growth of a root pushes regolith upwards normal to the ground surface. Upon decay, regolith directly above collapses into the void, thereby effecting a net flux downhill. As derived by Gabet, Reichman et al. (2003) the resulting flux is given by:

$$qs_{r\,dx} = \frac{z_{root} r \tau}{\sigma_{root}} sin\theta cos\theta \tag{1.4}$$

where z_{root} is the vertical depth of the root (O(0.3 m)); r is the root mass per unit area (O(4.4 kg m^{-2})); τ is the root turnover rate expressed as the annual below-ground production divided by the below-ground standing crop (O(0.56 yr^{-1})); ρ_{root} is the root bulk density (O(800 kg m^{-3})); and θ is the slope angle. This works out to a flux rate of 8.8 x 10^{-4} $sin\,\theta cos\,\theta$ m^2 yr^{-1} for hardwood forests. [Do we use this number for SHO or can Dave give us a better number?]

Tree-throw
Assume that trees fall over in random directions and carry regolith upwards in their rootballs such that at rest the root plate is perpendicular to slope and the regolith is sufficiently thick that the amount elevated is not a function of regolith thickness. Then following Gabet, Reichman et al. (2003), the volumetric flux of regolith in the horizontal x direction (Fig. 1) per unit width due to this process, qs_{ttx}, is given by:

$$qs_{ttx} = \frac{volume}{event} x \frac{distance}{event} x \frac{events}{area} x \frac{events}{time} \quad (1.5)$$

The volume per event must be obtained from a survey of trees in and near SHO, but studies in other regions quoted in Gabet, Reichman et al. (2003) indicate for hardwoods an average of 4 m^3 per event. The net downslope distance moved per event was derived by Gabet, Reichman et al. (2003) as

$$x_n = \frac{2}{\pi}(W + D) sin\theta \quad (1.6)$$

where W is the width of the root plate, D is the pit depth, and θ is the slope angle. Data from Norman, Schaetzl et al. (1995) for a deciduous forest in northern Michigan suggest that W=4 m and D= 0.7 m. [We should conduct a ground-based lidar survey of SHO and compute pit volumes; others have found an average of 4 m^3 and a range of 1-15 m^3]. Published uprooting rates as quoted in Gabet, Reichman et al. (2003) vary from 0.13—0.21 trees ha^{-1} yr^{-1} in a sugar maple and beech forest, 8 trees ha^{-1} yr^{-1} in a Northern hardwood forest, and 0.84 trees ha^{-1} yr^{-1} in an oak forest. We need to estimate this for the SHO region, but choosing a value of 4 yields a provisional flux equation for tree-throw:

$$qs_{ttx} = 4.8 \times 10^{-3} sin\theta \quad (1.7)$$

in units of m^2 yr^{-1}.

Rock Uplift Rate

The fourth term in (1.2) represents mass entering the control volume through its base. Rock uplift rates in the middle Atlantic states during the Cenozoic are on the order of a few meters per million years (Pazzaglia and Gardner 1994). This does not include the effects of crustal flexure from ice loading during the Pleistocene. [Use Peltier's predictions for glacial and post-glacial flexure]

Removal and Addition Due to Overland Flow

The last term in Equation (1.2) represents processes that transport sediment into and out of the control volume via overland flow. Because we will be linking this regolith module with a standard running-water sediment transport model, it makes sense to define E in terms of the mass conservation equation for overland sediment transport as, for example in Li (2008):

$$\frac{\partial(AC)}{\partial t}+\frac{\partial(QC)}{\partial x}-e(x,t)=0 \tag{1.8}$$

where A = cross sectional area of the overland flow (m^2), C = sediment concentration, Q = water discharge ($m^3\ s^{-1}$) [remember to convert from seconds to years], and e = net volumetric rate of erosion (positive) or deposition on the bed per unit width of flow ($m^2\ s^{-1}$). The term e represents the net rate of sediment added to or substracted from the control volume due to spatial gradients in the overland sediment flux. We consider two processes contributing to e:

$$e = DR + DF \tag{1.9}$$

where DR = the rate of regolith detachment by raindrop splash and DF = the rate of particle detachment due to the flow shear stress. Given the amount of leaf litter in SHO, DR will be ignored. Consistent with the European soil erosion model (EUROSEM) (Morgan, Quinton et al. 1998), DF is defined as:

$$DF = \beta w v_s (TC - C) \tag{1.10}$$

where β = flow detachment efficiency coefficient which depends upon the soil cohesion (1 for deposition, <1 for increasingly more cohesive soils) [We need to obtain field values for this at SHO], w is the width of flow, v_s is the particle settling velocity, TC is the transport capacity of the flow and C is the actual concentration of particles within the flow. E is equal to e per unit length in the x direction.

Discussion

It may be possible to eliminate some terms in (1.2) if their magnitudes are relatively small. The regolith production rate is not open for discussion since that would be throwing out the baby with the bathwater, but what about the relative flux magnitudes of creep, plant root decay, and tree-throw? Using values for the tunable parameters from the literature without much regard for forest type or climate yields:

$$qs_x = 0.005\tan\theta + 0.00088 sin\theta cos\theta + 4.8\times10^{-3} sin\theta \tag{1.11}$$

where θ is slope angle. Fixing the slope at 20 dg shows that creep and tree-throw are roughly equal and root decay is an order of magnitude smaller. This is not entirely fair, since I suspect there are biomechanical effects included in the creep data, but even so, it seems likely that root decay may be ignored. It also points out that tree-throw is relatively as important as freeze-thaw and other swelling processes [Folks--any comments?]

Do these equations produce a sensible landscape at dynamic equilibrium? At dynamic equilibrium the thickness of the regolith must remain temporally constant, and therefore whatever conversion of bedrock to regolith takes place per unit time must be balanced by a downslope gradient in regolith flux. Thus, after ignoring uplift and overland flow and substituting in the definition from Eqn. (1.3), Eqn. (1.1) becomes:

$$P_o e^{-\alpha h(x)} = \frac{\sigma_s}{\sigma_r}\frac{\partial qs_x}{\partial x} \tag{1.12}$$

This equation allows us to predict the spatial variation in soil thickness for a given gradient in regolith flux. Towards this end pits in the PAMAP lidar were filled and qs_x was computed from Eqn. (1.11) using the raster calculator in arcMAP. The results (Fig. 2) indicate that there are spatial gradients in flux, implying either that the regolith thickness must be varying to counterbalance those gradients OR that the system is not at steady state.

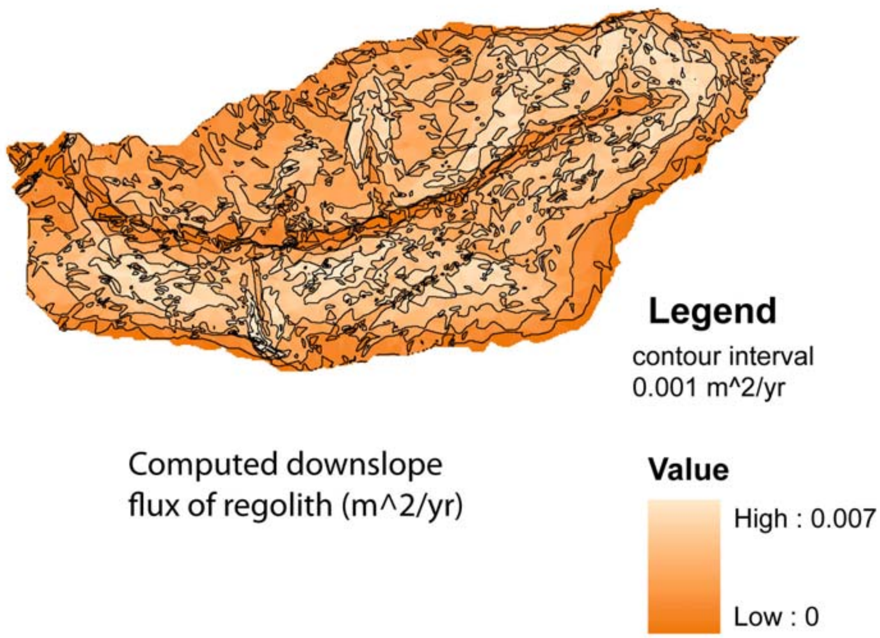

Figure 2. Computed regolith fluxes using Equation 1.10 and typical values of coefficients from the literature. The slopes were obtained from the lidar data.

Assuming the former and taking one transect down a hill along a flux line (more or less), fitting the fluxes with a quadratic equation in x (Figure 3), and differentiating that equation yields:

$$\frac{\partial qs_x}{\partial x} = 2p_1 x + p_2 \tag{1.13}$$

which, when substituted into Eqn. (1.12), yields:

$$h = -\frac{1}{\alpha} ln[Ax + B]$$
$$where \tag{1.14}$$
$$A = 2\frac{\sigma_s p_1}{P_o \sigma_r} \quad and \quad B = \frac{p_2 \sigma_s}{P_o \sigma_r}$$

For typical values of the variables, A = O(10^{-2}) and B = O(10^{0}). The shape of the resulting function indicates that the regolith thickness must increase exponentially downslope if the SHO is in steady state with the observed topography. [At least this is qualitatively consistent with observations---correct?]

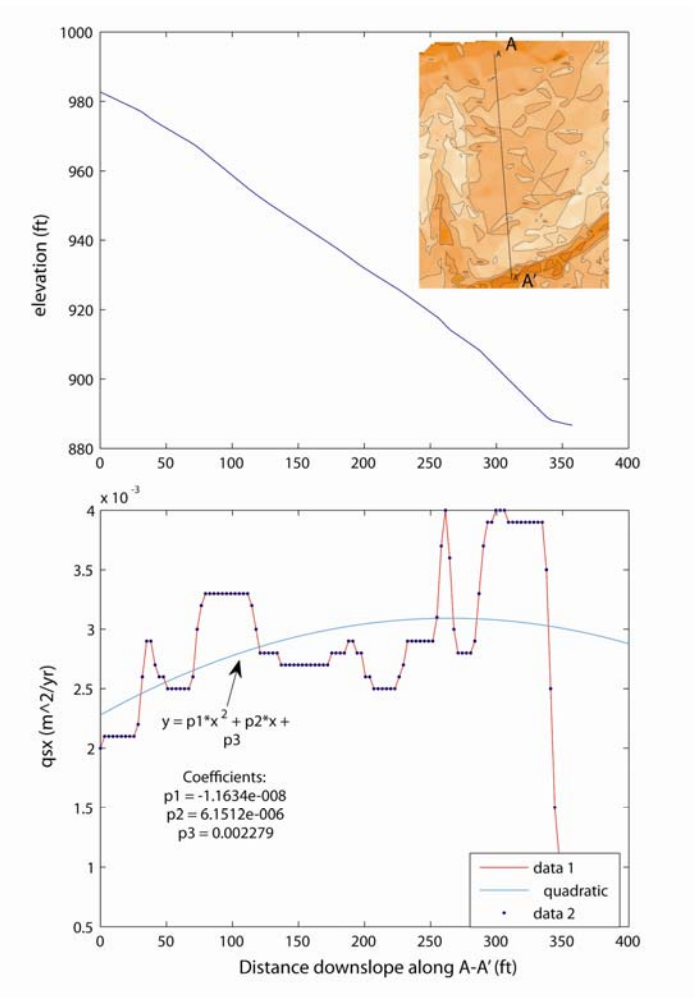

Figure 3. Predicted flux along a downslope transect on the north side of SHO.

Summary

In summary, PIHMSed needs to solve the following equation set to simulate landscape evolution:

$$
\begin{aligned}
&z = h + e \\
&\frac{\partial z}{\partial t} = -(\frac{\sigma_r}{\sigma_s} - 1)\frac{\partial e}{\partial t} - \frac{\partial qs_x}{\partial x} + \frac{\sigma_r}{\sigma_s}U - E \\
&\frac{\partial e}{\partial t} = -P_o e^{-\alpha h} \\
&qs_x = 0.005\tan\theta + 0.00088 sin\theta cos\theta + 4.8\times 10^{-3} sin\theta \\
&\theta = arctan(-\frac{dz}{dx})
\end{aligned} \tag{1.15}
$$

where U is specified and E is obtained from the overland sediment flux module, as for example in (Li 2008). Initial conditions for z, h, and e also must be specified. The boundary conditions are no lateral sediment flux at the drainage divide.

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